Numerical Approaches of Pricing European Options in the Cox-Ross-Rubinstein Models

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Objectives

1. Understanding option pricing in Finance.
2. Understanding fundamental mathematical theories in Probability.
3. Experiencing to get the result based on the theories in Finance and Probability.
4. Obtaining knowledge to get tables, graphs, and figures from data using Python programming language.

Mathematical Model

The $N-$period binomial model uses a pair of integers (n, j) to indicate each node in the tree, with $n = 0, 1, \ldots, N$ and $j = 0, 1, \ldots, n$. Over one period of time, an origin node (n, j) leads to node $(n + 1, j + 1)$ with probability p or leads to node $(n+1, j)$ with probability $1-p$. The index j counts the number of up changes to that time, so nj is the number of down changes. The stock value at each time step is therefore given by:

$$S_t = S_{t-1} U^j D^{n-j}. \tag{1}$$

There are (n, j) paths that lead to node (n, j) so the probability of going from price S_{t-1} to price $S_t = S_{t-1} U^j D^{n-j}$ is:

$$p_{(n,j)} = \binom{n}{j} p^j (1-p)^{n-j}. \tag{2}$$

Then the option price f for N-period binomial model is calculated as:

$$\text{option price} = \exp(-rT) \sum_{j=0}^{N} p(n, j) f(S_0 U^j D^{N-j}). \tag{3}$$

Introduction

The Cox-Ross-Rubinstein (CRR) market mode is used to price European and American Options without complex elements, including dividends, stocks, and stock indexes paying a continuous dividend yield, futures, and currency options. The model is an elegant, simple, but strong model to explain the general economic intuition behind option pricing and its principal techniques. In the paper, the CRR model's numerical elements and equations are indicated, and a practical event is examined to demonstrate the application of the model in the financial market. To make it easier to understand, figures, including tables and graphs, are also included to visualize and simplify the model and output data.

Practical Model

The Python program uses TSLA stock getting from the Nasdaq website, the scope is within one month of June 2021 (from June 1 to June 29). Number of time steps n = 20 since there are only 20 days in June 2021 that are updated with stock value.

Date	Stock	Input Data	
6/29/2021	680.76	T	1
6/28/2021	688.72	r	0.02
6/25/2021	671.87	**Output Data**	
6/24/2021	679.82	p	0.946
6/23/2021	656.57	n	20
6/22/2021	623.71	u_avg	1.022
6/21/2021	620.83	d_avg	0.984
6/18/2021	623.31	Euro Call	454.904
6/17/2021	616.6	Euro Put	0
6/16/2021	604.87		
6/15/2021	599.36		
6/14/2021	617.69		
6/11/2021	609.89		
6/10/2021	610.12		
6/9/2021	598.78		
6/8/2021	603.59		
6/7/2021	605.13		
6/4/2021	599.05		
6/3/2021	572.84		
6/2/2021	605.12		
6/1/2021	620		

Figure 7: TSLA in June 2021 and the Input-Output data summary.

Binomial Pricing Model with Two Steps

We consider the basic concept for the pricing model with step two:

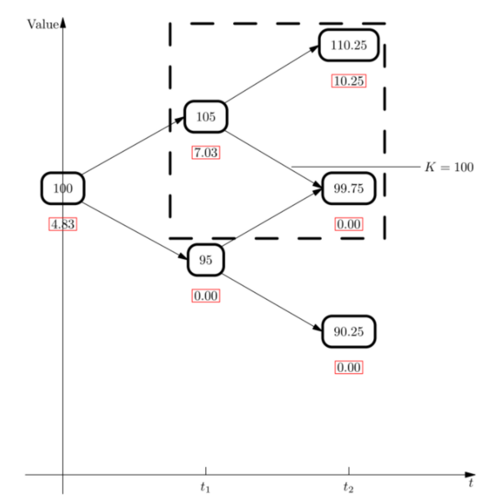

Figure 1: Basic Binomial Model for the European call option

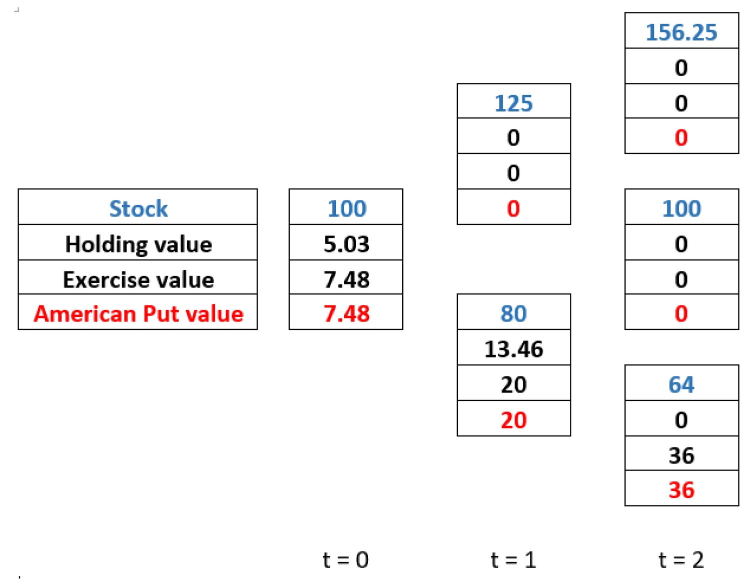

Figure 2: Two-period binomial model example for the European call option. The values of the security in each node are in blue. The values of the option in each node are in red.

Numerical Results and Tables

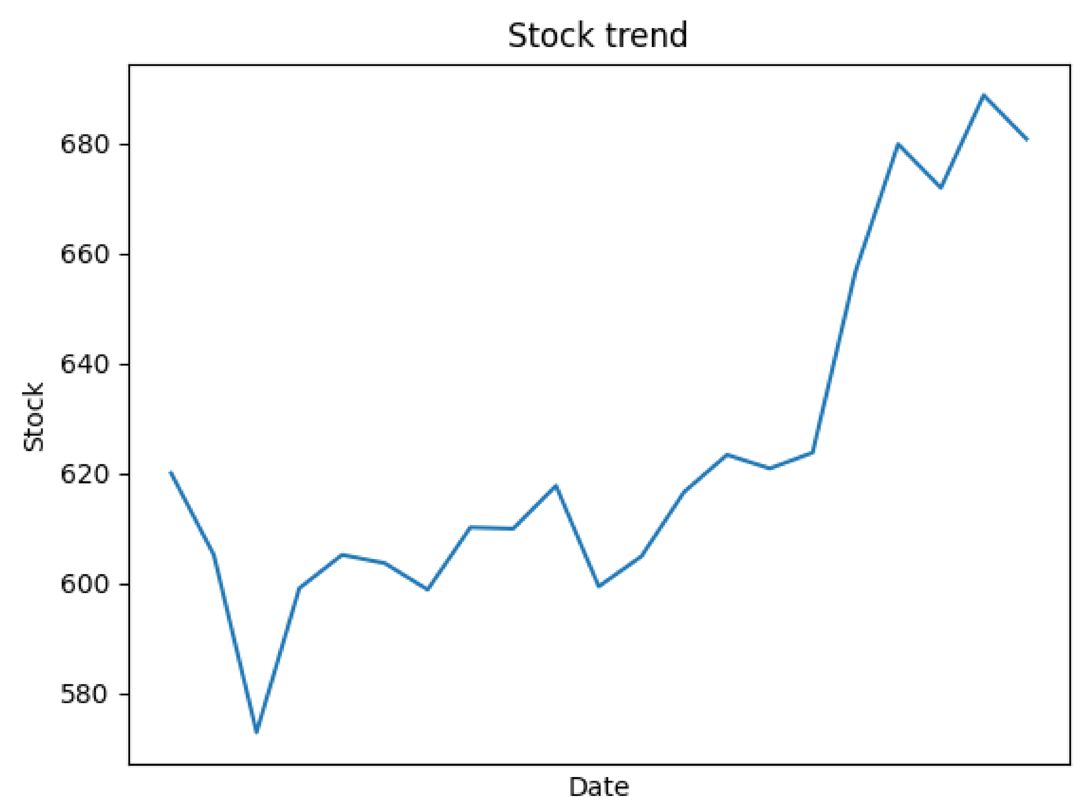

Figure 3: Stock Trend over June 2021.

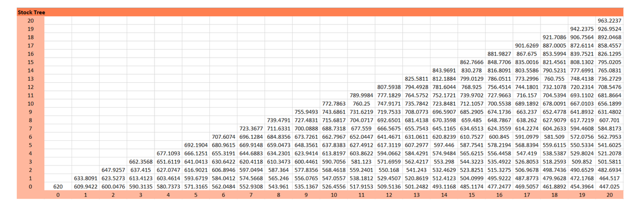

Figure 4: Stock Tree.

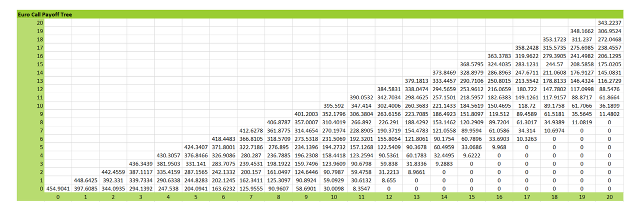

Figure 5: European Call Option Tree.

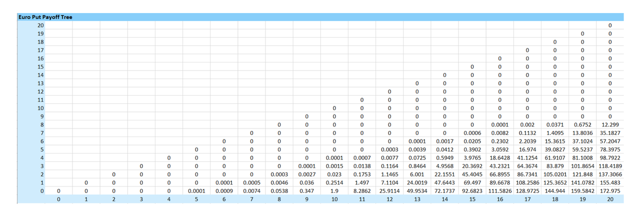

Figure 6: European Put Option Tree.

References

[1] John C Cox, Stephen A Ross, and Mark Rubinstein. Option pricing: A simplified approach. *Journal of financial Economics*, 7(3):229–263, 1979.

[2] Steven R Dunbar. *Mathematical Modeling in Economics and Finance: Probability, Stochastic Processes, and Differential Equations*, volume 49. American Mathematical Soc., 2019.

Conclusion

- The binomial model that we investigated is using an iterative approach utilizing multiple periods.
- It is the widely adopted mathematical formula for valuing several kinds of options in economics.
- Analyzing the model gives us insight into a stream of stocks or assets and estimates their values. Using Python code gave the methodology to explore the model and how to assess the pricing value.

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